

Intermediate Scales in SUSY SO(10), b - τ Unification, and Hot Dark Matter Neutrinos¹

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Abstract

Considerations of massive neutrinos, baryogenesis as well as fermion mass textures in the grand unified theory framework provide strong motivations for supersymmetric(SUSY) SO(10) as the theory beyond the standard model. If one wants to simultaneously solve the strong CP problem via the Peccei-Quinn mechanism, the most natural way to implement it within the framework of the SUSY SO(10) model is to have an intermediate scale (v_{BL}) (corresponding to B-L symmetry breaking) around the invisible axion scale of about 10^{11} - 10^{12} GeV. Such a scale is also desirable if ν_τ is to constitute the hot dark matter (HDM) of the universe. In this paper, we discuss examples of superstring inspired SUSY SO(10) models with intermediate scales that are consistent with the low energy precision measurements of the standard model gauge couplings. The hypothesis of b - τ unification which is a successful prediction of many grand unified theories is then required of these models and the resulting prediction of b -quark mass is used as a measure of viability of these schemes.

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Detailed analysis of a model with a $v_{BL} \simeq 10^{11}$ GeV, which satisfies both the requirements of invisible axion and ν_τ as HDM is presented and shown to lead to $m_b \simeq 4.9$ GeV in the one-loop approximation.

1 Introduction

There are many reasons for the recent surge of interest in supersymmetric (SUSY) SO(10) models such as (i) a possibility to understand the observed patterns of fermion masses and mixings[1]; (ii) small non-zero neutrino masses[2, 3]; (iii) a simple mechanisms for baryogenesis[4], etc. The recent observation[5] that level two compactification of superstring models can also lead to SO(10) models with higher dimensional multiplets such as **45** and **54** that can help in grand unified theory (GUT) symmetry breaking has injected new enthusiasm to this field.

In all SO(10) models considered to date, one assumes a grand desert scenario between the TeV scale (corresponding to the electroweak symmetry as well as SUSY breaking) and the GUT scale, M_U , of order 10^{16} GeV. This scenario is of course required if the known low energy gauge couplings are to unify at M_U given the particle spectrum of the minimal supersymmetric standard model (MSSM)[6]

There are however reasons to entertain the possibility that a GUT model such as SO(10) should have an intermediate scale around $10^{11} - 10^{12}$ GeV corresponding to B-L symmetry breaking. The first one has to do with solving the strong CP problem via the Peccei-Quinn mechanism[7]. Since cosmological constraints require the PQ-symmetry breaking scale, v_{PQ} , around $10^{11} - 10^{12}$ GeV, it will fit naturally into an SO(10) model if a gauge subgroup of SO(10) such as $SU(2)_R \times U(1)_{B-L}$ breaks around the same intermediate scale, *i.e.*, $v_{B-L} \sim v_{PQ}$. Another reason that such model may be of interest has to do with the possibility that the tau neutrino with a mass of few electron volts may constitute the hot dark matter (HDM) component of the universe needed to fit the observations on the large scale structure with the successful big bang picture[8]. In the see-saw mechanism for neutrino masses predicted by a class of SO(10) models, this requires that there must be an intermediate scale corresponding to the $B - L$ symmetry breaking around $10^{11} - 10^{12}$ GeV, which is

of the same order as required for the invisible axion scenario. The fact that such an $SO(10)$ scenario emerges naturally in non-supersymmetric context has been known for some time[9].

As far as the neutrino mass alone is concerned, one could argue that an eV range mass for the tau neutrino could be obtained in the grand desert type $SO(10)$ models by judicious "dialing" down of the Yukawa coupling of the $\overline{\mathbf{126}}$ coupling to the matter spinors. This would of course not accomodate the invisible axion solution to the strong CP-problem. Moreover, in two interesting recent papers[10], it has been noted that at least for the small $\tan\beta$ case, this alternative may run into trouble with the hypothesis of bottom-tau mass unification[11], which is another successful prediction of grand unified theories. Of course one could abandon the $b-\tau$ unification hypothesis as in $SO(10)$ models which contain $\mathbf{126} + \overline{\mathbf{126}}$ multiplets (e.g. see Ref.[3]) or one could consider a large $\tan\beta$ scenario. But if we insist on a small $\tan\beta$, then an alternative that is available is to abandon the grand desert scenario and consider intermediate scale type $SO(10)$ models and see if it is consistent with the $b-\tau$ unification hypothesis. In this paper, we explore this possibility.

We first seek simple extensions of the minimal SUSY $SO(10)$ model which can support intermediate scales corresponding to B-L symmetry breaking consistent with the gauge coupling unification. We will assume that supersymmetry is an exact symmetry above the weak scale, as is generally believed. Several such models have already been discussed in the literature[12, 13]. In our work, we will assume that the particle spectrum below the Planck scale is of the type dictated by recent level two Kac- Moody schemes[5], so that they contain three $\mathbf{16}$ dim. spinors corresponding to three generations of matter fields, a number of $\mathbf{10}$, $\mathbf{16}_H + \overline{\mathbf{16}}_H$, $\mathbf{45}$ and $\mathbf{54}$ dimensional fields. This is one of the respects in which our work differs from earlier works in Ref.[12]. For this case, by appropriate adjustments of the particle spectrum, we have found several new classes of intermediate scale models. If we choose a value of the

QCD coupling $\alpha_{3c} \geq .115$, only one class of these models is singled out as preferable to the others in the one loop approximation. Since the two loop corrections to these results are not that drastic, we choose to do a more detailed analysis with this model. We then impose the additional requirement that the Yukawa couplings corresponding to the bottom quark and the tau lepton unify at M_U and use the prediction for the bottom quark mass as an indicator of the viability of a given scenario. We are able to find one scenario which has $v_{BL} \simeq 10^{11}$ GeV as desired, with a prediction for $m_b \simeq 4.9$ GeV in the one loop approximation, which we believe is phenomenologically acceptable[14]. This is the main result of our paper.

We have organized this paper as follows: in Sec. 2, we discuss examples of SUSY SO(10) models, where intermediate B-L symmetry breaking scales can arise, consistent with gauge coupling unification; in Sec. 3, we discuss the restrictions of b- τ Yukawa unification in these models; in Sec. 4, we discuss how neutrino masses are understood in this class of model; in Sec. 5, we present our concluding remarks.

2 Gauge Coupling unification and the Intermediate Scale for B-L Symmetry Breaking

It is well-known that if one assumes exact supersymmetry above the TeV scale and the particle spectrum of the MSSM, there is no room for an intermediate scale consistent with gauge coupling unification. On the other hand, there exist several examples[12, 13] where changing the spectrum can lead to a variety of possibilities for intermediate scales corresponding to $SU(2)_R \times U(1)_{B-L}$ symmetry breaking. Our goal is to seek an intermediate scale around 10^{10} - 10^{12} , motivated by the allowed scale for the invisible axion solution to the strong CP problem and an HDM ν_τ . In our analysis of gauge coupling unification, we will first use one-loop beta function to get a rough idea about the nature of intermediate scales. We will then pick out

the scenario which has the best chance of fulfilling our requirements and do a two loop analysis for the gauge coupling evolution to find the more exact value of the intermediate scale. At the two-loop level, there is a top-quark contribution to the beta-function. In our calculation we will ignore this for simplicity of calculation. The relevant evolution equations for the gauge couplings are:

$$\frac{d\alpha_i}{dt} = \frac{b_i}{2\pi}\alpha_i^2 + \sum_j \frac{b_{ij}}{8\pi^2}\alpha_i^2\alpha_j, \quad (1)$$

where $i=1, 2, 3$, between $M_Z \leq \mu \leq M_R$ and denote the $U(1)_Y$, $SU(2)_L$, $SU(3)_c$ -symmetries respectively, whereas $i=1, 2, 3, 4$, for $M_R \leq \mu \leq M_U$ and denote the $U(1)_{B-L}$, $SU(2)_L$, $SU(2)_R$, $SU(3)_c$ -symmetries respectively.

Before presenting the detailed results, let us discuss the one loop evolution equations to get an idea about the nature of the models that can support an intermediate scale.

$$2\pi[\alpha_i^{-1}(M_Z) - \alpha_U^{-1}(M_U)] = b_i R + b'_i(U - R) \quad (2)$$

where we have denoted $U = \ln \frac{M_U}{M_Z}$ and $R = \ln \frac{M_R}{M_Z}$; b'_2 and b'_3 stand for the values for the $SU(2)_L$ and $SU(3)_c$ beta function coefficients above the M_R scale and $b'_1 = \frac{2}{5}b_{B-L} + \frac{3}{5}b_{2R}$. Using these one-loop equations, several solutions were found in Ref.[12] where one can have $M_R \simeq 10^{11} - 10^{12}$ GeV; Since we are interested in solutions with similar values for M_R , let us mention the two solutions found there:

Solution A: The Higgs multiplets above M_R have the $U(1)_{B-L} \times SU(2)_L \times SU(2)_R \times SU(3)_c$ (called G_{LR} in what follows): one of $(2, 1, 3, 1) + (-2, 1, 3, 1)$; $(0, 3, 1, 1)$; $(0, 1, 1, 8)$ each and two of $(0, 2, 2, 1)$.

Solution B: In this case, the Higgs multiplets above M_R have G_{LR} transformation properties: one of each of the following: $(2, 1, 3, 1) + (-2, 1, 3, 1)$; $(1, 1, 2, 1) + (-1, 1, 2, 1)$; $(0, 2, 2, 1)$; $(0, 3, 1, 1)$ and $(0, 1, 1, 8)$.

Note that both these solutions require the existence of the $\mathbf{126} + \overline{\mathbf{126}}$ pair at the GUT scale; therefore they cannot emerge from simple superstring models with either Kac-Moody level one or two [5]. We therefore seek solutions that do not involve these multiplets. We have found six solutions using the method described in Ref[15]. One of them is the one already found by Deshpande, Keith and Rizzo[13]. They are all characterized two integers (n_H, n_X) , where n_H represents the number of $(0, 2, 2, 1)$ multiplets and n_X represents the number of $(1, 1, 2, 1) + (-1, 1, 2, 1)$ multiplet pairs above the scale M_R . Note that these scenarios necessarily involve D-parity breaking[16]. Below we give the one and two loop beta function coefficients b_i and b_{ij} for different mass ranges for these cases and in table I, we list the solutions.

$$\begin{aligned}
\text{(i) For } M_{SUSY} \leq \mu \leq M_R, \quad & \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} \frac{33}{5} \\ 1 \\ -3 \end{pmatrix}, \quad b_{ij} = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix}; \\
\text{(ii) For } M_R \leq \mu \leq M_U, \quad & \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \begin{pmatrix} 6 + \frac{3}{2}n_X \\ n_H \\ n_H + n_X \\ -3 \end{pmatrix}, \quad b_{ij} = \begin{pmatrix} 7 + \frac{9}{4}n_X & 9 & 9 + \frac{9}{2}n_X & 8 \\ 3 & 18 + 7n_H & 3n_H & 24 \\ 3 + \frac{3}{2}n_X & 3n_H & 18 + 7n_H + 7n_X & 24 \\ 1 & 9 & 9 & 14 \end{pmatrix}.
\end{aligned}$$

Note that the light $(0, 2, 2, 1)$ Higgses originate from 10-dim. $\text{SO}(10)$ representation and the light Higgs pairs of $(-1, 1, 2, 1) + (1, 1, 2, 1)$ originate from 16-dim. $\text{SO}(10)$ representation.

In order to discuss the implications of these equations, we use the following values of $\alpha_i(M_Z)$ [17]

$$\begin{aligned}
\alpha_1^{-1}(M_Z) &= 58.97 \pm 0.05, \\
\alpha_2^{-1}(M_Z) &= 29.62 \pm 0.04,
\end{aligned} \tag{3}$$

$$\alpha_{3c}(M_Z) = 0.120 \pm 0.013. \quad (4)$$

For a given model, the value of the intermediate scale M_R depends mainly upon $\alpha_{3c}(M_Z)$. We display this dependence in Fig. 1 using one-loop renormalization equations, where only the mean values of $\alpha_i(M_Z)$ ($i=1,2$) are used. From Figure 1, it is clear that only for the two models (V) and (VI) one can have intermediate scales around 10^{11} GeV, in which we are interested. For those two models, we give the two-loop running of the three gauge couplings from $M_{SUSY} \approx m_t$ scale to M_U , in Figure 2 and 3. These two-loop results are little changed from the one-loop results. We also wish to note that between M_Z and m_t , we use the standard model beta functions and then treat the top mass as a threshold correction[18]. We have not included any other threshold corrections in our calculations.

3 Constraint of b- τ Unification

In this section, we will explore whether it is possible to achieve b- τ mass unification in the class of intermediate scale scenarios discussed in the previous section. Let us first give a qualitative overview of the issues involved here. It is well-known that Yukawa couplings for the bottom quark (h_b) and the tau lepton (h_τ) evolve differently above the weak scale. There are two classes of diagrams that control their evolution: (i) one class involving the virtual gauge bosons and (ii) a second class involving virtual Higgs bosons. We will keep their effects up to one-loop for all the quarks and display the equations in terms of $Y_i \equiv \frac{h_i^2}{4\pi}$, where $i = t, b, \tau$:

$$\frac{dY_t}{dt} = \frac{Y_t}{2\pi} \left[6Y_t + Y_b - \sum_i c_i^{(t)} \alpha_i \right], \quad (5)$$

$$\frac{dY_b}{dt} = \frac{Y_b}{2\pi} \left[Y_t + 6Y_b + Y_\tau - \sum_i c_i^{(b)} \alpha_i \right], \quad (6)$$

$$\frac{dY_\tau}{dt} = \frac{Y_\tau}{2\pi} \left[4Y_\tau + 3Y_b - \sum_i c_i^{(\tau)} \alpha_i \right], \quad (7)$$

where $i = 1, 2, 3$ denote $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$ respectively, and $c_i^{(t)} = (13/15, 3, 16/3)$; $c_i^{(b)} = (7/15, 3, 16/3)$; $c_i^{(\tau)} = (9/5, 3, 0)$. We also note that we have redefined the $U(1)_Y$ gauge coupling so that the new $U(1)_Y$ -charges, \tilde{Y} are given by $\tilde{Y} = \sqrt{\frac{3}{5}}Y$, Y being the canonically assigned $U(1)_Y$ -charge.

It is well-known [19] that if all Yukawa couplings were small, then simple closed form solutions relating the Yukawa couplings at different mass scales can be written down. In particular one can obtain

$$\frac{h_b(M_Z)}{h_\tau(M_Z)} = \frac{h_b(M_U)}{h_\tau(M_U)} \left(\frac{\alpha_1(M_Z)}{\alpha_1(M_U)} \right)^{-\frac{c_1^{(b)} - c_1^{(\tau)}}{2b_1}} \left(\frac{\alpha_{3c}(M_Z)}{\alpha_{3c}(M_U)} \right)^{-\frac{c_3^{(b)}}{2b_3}} \equiv \frac{h_b(M_U)}{h_\tau(M_U)} R_g^U. \quad (8)$$

The b - τ unification scenario implies $h_b(M_U) = h_\tau(M_U)$. If we ignore the effects of the $U(1)_Y$ -coupling, we can easily see that since $\alpha_3(M_Z) > \alpha_3(M_U)$ and $b_3 < 0$, $h_b(M_Z) > h_\tau(M_Z)$ as needed. Numerically, evaluating the right-hand side of Eq. (8), one finds that $h_b(M_Z)/h_\tau(M_Z) = \frac{m_b}{m_\tau}(M_Z) \simeq R_g^U$ exceeds the experimental value (denoted by R_{expt}) by about 30%.² Fortunately, experimental indications of a large top quark mass already implies that h_t effects cannot be ignored in the evolution of h_b ; Moreover, for large $\tan\beta$ (where $\tan\beta = v_u/v_d$ with $v_u = \langle H_u^0 \rangle$ and $v_d = \langle H_d^0 \rangle$ for MSSM), effects of h_b and h_τ cannot be ignored either. This restores agreement between GUT scale b - τ unification and known masses of the b and τ . In a sense, given the free parameter $\tan\beta$, b - τ unification in a simple grand desert type GUT model is not very constraining and an arbitrary choice of h_t and

²We remind the reader that in extrapolating from $m_b(m_b)$ and $m_\tau(m_\tau)$ to their values at M_Z , we have used the three-loop QCD and one-loop QED beta function effects[20].

$\tan\beta$ helps in making $b - \tau$ unification an experimental success. In fact for the case of small $\tan\beta$, this can be seen explicitly from the following formula[10]:

$$R_{b/\tau}(M_Z) = R_g^U \times e^{-A_U}, \quad (9)$$

where

$$A_U = \frac{1}{16\pi^2} \int_{t_Z}^{t_U} h_t^2(t) dt, \quad (10)$$

It is clear that by adjusting h_t , the value of A_U can be made bigger than one producing the desired suppression.

Suppose that we now go beyond the grand desert scenario and require, as we do here, that there is a gauge intermediate scale, M_R corresponding to $SU(2)_R \times U(1)_{B-L}$ breaking. Several new features emerge:

- (i) The nature of the Yukawa couplings and their evolutions change due to the presence of new symmetries and new particles above M_R .
- (ii) The evolution of gauge couplings change due to the same reasons.
- (iii) Finally between M_R and M_U , $b - \tau$ mass unification implies that there is cancellation between h_t and h_N (where h_N is Dirac type coupling of the neutrinos) in the evolution equation for h_b so that the correct experimental value is not guaranteed merely by the choice of h_t [10].

In this case, one can write:

$$R_{b/\tau}(M_Z) = R_g^I \cdot R_g^{IU} \cdot e^{-A_I - A_{IU}} \quad (11)$$

where R_g^I and R_g^{IU} represent respectively the gauge contributions between M_Z and M_R and between M_R and M_U and similarly the A_I and A_{IU} represent the Yukawa coupling contributions in the two ranges. The expression for A_I is easily obtained from Eq. (10) by replacing the upper limit by t_R ; in order to get the expression for A_{IU} , we need the evolution equations for the Yukawa couplings above M_R which are given below.

Note that in contrast with the case discussed in [10], where there was no gauge intermediate scale, we have now a new possibilities to bring $R_{b/\tau}(M_Z)$ into agreement with data. The simplest way is to assume that above the intermediate scale M_R , the QCD beta function coefficient b_3 becomes zero, so that α_{3c} becomes flat; then R_g^{IU} is given purely by the $U(1)_Y$ evolution and one has roughly $R_g^{IU} \approx 1$ and the value of $R_{b/\tau}(M_Z)$ is determined by the value of $\alpha_{3c}(M_R)$ which is always bigger than $\alpha_{3c}(M_U)$. This leads to $R_g^I < R_g^U$ which without any help from the top Yukawa coupling can lead to agreement with R_{expt} . It is easy to see that this behaviour can occur if there is a color octet with mass at M_R . It however turns out that in simple string inspired models, it is difficult to get such a light octet at M_R . We will therefore have to be content with the situation where there are no color fields at M_R (see table I) and see numerically what the prediction for $R_{b/\tau}(M_Z)$ is.

Another point that is important for our discussion is the embedding of the MSSM Higgs doublets into the GUT multiplets. In the case of a single **10** SO(10) model (in the absence of any **126**+ $\overline{\mathbf{126}}$ multiplets) both the MSSM Higgs multiplets are part of this **10** multiplet which leads to complete Yukawa unification (*i.e.*, $h_t = h_b = h_\tau$ at M_U) [21]. While such models are quite elegant, a complete understanding of why $\tan\beta$ is large in this case becomes difficult. We will therefore focus on the simple class of models where there are two **10**-Higgses at the GUT scale. In general, the Higgs doublets H_u, H_d of MSSM would be some linear combinations of the doublets in **10**'s. But we will adopt a simple doublet-triplet splitting pattern such that for $\mu < M_R$, H_u and H_d of MSSM arise from different **10**'s. That such a situation is possible has been shown in Ref. [22]. This assumption though not crucial for our conclusions helps to simplify our calculations.

In the case where only one $\phi(0,2,2,1)$ exists in the intermediate scale, the Yukawa sector of the Lagrangian of the models is given by

$$\mathcal{L}_Y = h_Q Q^T \tau_2 \phi Q^c + h_L L^T \tau_2 \phi L^c. \quad (12)$$

The equations used for Yukawa couplings are

$$\frac{dY_Q}{dt} = \frac{Y_Q}{2\pi} \left[7Y_Q + Y_L - \sum_i c_i^{(Q)} \alpha_i \right], \quad (13)$$

$$\frac{dY_L}{dt} = \frac{Y_L}{2\pi} \left[3Y_Q + 5Y_L - \sum_i c_i^{(L)} \alpha_i \right], \quad (14)$$

where $Y_Q = \frac{h_Q^2}{4\pi}$ and $Y_L = \frac{h_L^2}{4\pi}$; $i = 1, 2, 3, 4$ denote $U(1)_{B-L}$, $SU(2)_L$, $SU(2)_R$, and $SU(3)_C$ respectively; $c_i^{(Q)} = (1/6, 3, 3, 16/3)$; $c_i^{(L)} = (3/2, 3, 3, 0)$.

For reasons stated above, we will be interested in the case where two (0,2,2,1)-Higgses appear at M_R . Then the Yukawa sector of the Lagrangian at M_R is then given by:

$$\mathcal{L}_Y = h_{Q_1} Q^T \tau_2 \phi_1 Q^c + h_{Q_2} Q^T \tau_2 \phi_2 Q^c + h_{L_1} L^T \tau_2 \phi_1 L^c + h_{L_2} L^T \tau_2 \phi_2 L^c. \quad (15)$$

The corresponding equations for Yukawa coupling evolution are:

$$\frac{dY_{Q_1}}{dt} = \frac{Y_{Q_1}}{2\pi} \left[7Y_{Q_1} + 4Y_{Q_2} + Y_{L_1} - \sum_i c_i^{(Q_1)} \alpha_i \right], \quad (16)$$

$$\frac{dY_{Q_2}}{dt} = \frac{Y_{Q_2}}{2\pi} \left[4Y_{Q_1} + 7Y_{Q_2} + Y_{L_2} - \sum_i c_i^{(Q_2)} \alpha_i \right], \quad (17)$$

$$\frac{dY_{L_1}}{dt} = \frac{Y_{L_1}}{2\pi} \left[3Y_{Q_1} + 5Y_{L_1} + 4Y_{L_2} - \sum_i c_i^{(L_1)} \alpha_i \right], \quad (18)$$

$$\frac{dY_{L_2}}{dt} = \frac{Y_{L_2}}{2\pi} \left[3Y_{Q_2} + 4Y_{L_1} + 5Y_{L_2} - \sum_i c_i^{(L_2)} \alpha_i \right]. \quad (19)$$

For the models considered in this paper, $c_i^{(Q_1)} = c_i^{(Q_2)} = c_i^{(Q)}$ and $c_i^{(L_1)} = c_i^{(L_2)} = c_i^{(L)}$. We assume that the MSSM Higgs doublets H_u and H_d are embedded in ϕ_1 and ϕ_2 respectively.

These equations are supplemented by the one-loop evolution equations for the

gauge couplings already described in the previous section³. We can now write down the expression for A_{IU} in Eq. (11):

$$A_{IU} = \frac{1}{16\pi^2} \int_{t_R}^{t_U} [4(h_{Q_1}^2(t) - h_{L_1}^2(t))] dt \quad (20)$$

In writing the above equation, we have ignored the bottom and tau Yukawa coupling effects which come from h_{Q_2} and h_{L_2} . We wish to point out that there is an extra factor of four in the exponent in the above equation relative to the same equation in the grand desert scenario[10], which will tend to magnify our contribution somewhat. To see the effect numerically, we follow the procedure given below: For a given model, using mean values for $\alpha_{1Y}(M_Z)$ and $\alpha_{2L}(M_Z)$, choose an intermediate scale which yields $\alpha_{3c}(M_Z)$ in the input ranges of $\alpha_{3c}(M_Z)$. This also fixes $\alpha_U(M_U)$ as well as M_U . A given $\tan\beta$ together with the experimental values for top quark and τ masses can determine the initial values for the Yukawa couplings, $Y_{Q_i}(M_U)$ and $Y_{L_i}(M_U)$. Then the Yukawa and gauge coupling constants are numerically extrapolated, and the bottom quark mass is predicted.

We have scanned a large region in the $\tan\beta$ - M_R space for small $\tan\beta$ so that the effects of h_b and h_τ can be ignored in the Yukawa coupling evolution equations. We find that the best case scenario which is also physically interesting from the point of view of invisible axion and ν_τ as HDM emerges when $M_R \simeq 10^{11}$ GeV; $\tan\beta \simeq 1.7$. In this case, $h_t(M_U) \simeq 3.54$, $\alpha_U^{-1} \simeq 23.64$ and $M_U \simeq 3.43 \times 10^{15}$ GeV. The prediction for the bottom quark mass (pole mass) is $m_b \simeq 4.9$ GeV. All other choices of $\tan\beta$ and M_R lead to larger values of m_b . We realize that this value of m_b may be somewhat on the high side but we wish to note that we have only used the one loop equations for the Yukawa coupling evolution and in any case such a value is strictly not ruled out[14]. The evolution of the Yukawa couplings in this case are

³ Note that in the above evolution equations only the contributions from the **10** Higgs couplings to matter spinors are present. For models where $\overline{\mathbf{126}}$ contributions exist, see Ref. [23]

depicted in Fig. 4.

4 Tau Neutrinos as Hot Dark Matter and Intermediate Scale SO(10) Models

In the previous sections, we established the existence of simple SO(10) models with intermediate B-L symmetry breaking scales consistent with low energy data on gauge couplings with a reasonable prediction for m_b/m_τ . Let us now discuss whether such models can indeed lead to a tau neutrino mass in the 5 to 7 electron Volt range as required for it to be the HDM component of the universe. The reason such a discussion is called for is the following. A notable feature of the models we have discussed is the absence of **126**+ $\overline{\mathbf{126}}$ Higgs multiplets, which are needed in the implementation of see-saw mechanism for neutrino masses. Therefore, the existence of a B-L breaking scale around 10^{11} - 10^{12} GeV does not necessarily guarantee m_{ν_τ} in the several eV mass range. We will show in this section, that it is possible to use a generalized see-saw mechanism[24] such that even without the presence of **126**+ $\overline{\mathbf{126}}$ representations, one can get see-saw-like formula for light neutrino masses. To see our proposal in detail, let us denote the Higgs-like **16**+ $\overline{\mathbf{16}}$ multiplets by $\Psi_H \oplus \bar{\Psi}_H$, and matter spinors by Ψ_a . Let us introduce 3 gauge singlet fields, S_a . The part of the superpotential relevant for neutrino masses is⁴

$$W_\nu = h_{ab}^{(1)} \Psi_a \Psi_b H_1 + f_{ab} \Psi_a \bar{\Psi}_H S_b + M_{ab} S_a S_b. \quad (21)$$

Recall that H_1 is the **10**-dim. Higgs multiplet which leads to the H_u -type higgs

⁴ These new couplings introduced are assumed to be sufficiently small so as not to effect the evolution equations for the Yukawa couplings.

doublet of MSSM and is therefore responsible for the Dirac mass of the neutrinos. The resulting mass matrix involving ν, N , and S is given in the basis $\{\nu_a, N_a, S_a, (a=1,2,3)\}$ by

$$M_\nu = \begin{pmatrix} 0 & h^{(1)}v_u & 0 \\ h^{(1)}v_u & 0 & f v_R \\ 0 & f^T v_R & M \end{pmatrix}, \quad (22)$$

Note that $h^{(1)}, f$, and M are 3×3 matrices. It is clear that if we ignored all generation mixings then $h^{(1)}, f$, and M will be diagonal and the mass of a -th light Majorana neutrino will be given by

$$m_{\nu_a} \sim \frac{(h_a^{(1)}v_u)^2 M_a}{f_a^2 v_R^2} \quad (23)$$

If we assume that $f_a v_R \sim M_a$, then the familiar see-saw formula results and for $v_R \sim 10^{11}$ to 10^{12} GeV, m_{ν_τ} is in the eV range. Let us be clear that unlike the simple 2×2 see-saw models, one cannot make definite predictions for neutrino masses and mixings due to the presence of arbitrary singlet mass M_a . Also note that the value of f_a should not be too much smaller than one since in our discussion in Sec. 2 and Sec. 3, we have assumed that the right-handed neutrino contributes to renormalization group equations for $M_N \geq v_R$. Looking at the formula Eq. (23), one might think that regardless of the value of v_R , one might get an eV range mass for ν_τ by simply adjusting M_{N_3} . This is however not true; if $M_a \gg f_a v_R$, the mass of the heavy right-handed neutrino (say N_τ) becomes $(f_a v_R)^2/M_a$, which is much less than $f_a v_R$. In this case, the contribution of N_τ to renormalization group evolution of Yukawa couplings will start much below v_R , contrary to what is assumed in the discussion of b- τ unification. Thus, our discussion of b- τ unification essentially restricts $M_a \approx f_a v_R$; as a result, one recovers the usual 2×2 see-saw formula for ν_τ mass and a few eV ν_τ goes with a $v_R \approx 10^{11} - 10^{12}$ GeV.

5 Conclusions and Outlook

To summarize, we have analyzed the possibility that supersymmetric SO(10) models have an intermediate scale around 10^{11} or 10^{12} GeV so that they can naturally accomodate the invisible axion mechanism to solve the strong CP problem and also provide room for the tau neutrino to have a mass in the range of 6 to 7 eV so that it can constitute the HDM component of the universe. We have tried to stay within the constraint of a superstring inspired particle spectrum. We have found a scenario which has the above property. We have then analyzed whether the desirable property of $b - \tau$ mass unification holds in this scheme in the small $\tan\beta$ region. We find the answer to be yes provided we accept a value for the pole mass value for m_b of 4.9 GeV (within the framework of a one-loop analysis). The spectrum at the intermediate scale needed is generated without complicated fine tuning once one realizes that the model breaks D-parity at the GUT scale due to the presence **45** multiplet having vacuum expectation value along the **(1,1, 15)** direction. Finally we wish to note that the model has enough flexibility that one can extend it to understand the fermion mass textures (using for example the methods of Ref.[22]).

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Table I

Model	(n_H, n_X)
I	(1, 2)
II	(1, 3)
III	(1, 4)
IV	(2, 3)
V	(2, 4)
VI	(2, 5)

Table Caption: In this table, we give the Higgs particle contents at the scale M_R that define the different models; The symbols (n_H, n_X) denote the numbers of (0, 2, 2, 1) and (1, 1, 2, 1)+(-1, 1, 2, 1) multiplets respectively.

Figure Caption

Figure 1: The values of the intermediate mass scale M_R for different scenarios as a function of the $\alpha_{3c}(M_Z)$ in the one-loop approximation. The case II corresponds

to a vertical line where all values of intermediate scales are allowed in the one-loop approximation, since the evolution equation becomes independent of M_R .

Figure 2: Evolution of gauge coupling in the two-loop approximation for the scenario V.

Figure 3: Evolution of the gauge couplings in the two-loop approximation for the scenario VI.

Figure 4: (a) Evolutions of the gauge couplings in the one-loop approximation and (b) evolution of the Yukawa couplings h_t (labelled A), h_N (labelled B), and the ratio h_b/h_τ (labelled C) for the scenario V which is physically most interesting.

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